# Lagrangian Formalism and Graviton Quantization in the Grand Unified Field Theory of Asymmetry

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**Abstract**

We formulate a complete field-theoretic action for the Grand Unified Field Theory of Asymmetry (GUFT). Spacetime emerges as a coherent phase of the Dark Energy Quantum Field (DEQF), while the Graviton Quantum Field (GQF) encodes quantised curvature. We: (i) present a covariant Lagrangian whose classical limit reproduces Einstein gravity plus a cosmological constant, (ii) specify an explicit DEQF order parameter and potential, (iii) canonically quantise the graviton field over the DEQF background, and (iv) impose a Planck-scale saturation rule that renders loop integrals ultraviolet-finite. In the semiclassical limit the construction reduces to General Relativity + Standard Model; at high energies it predicts distinct signatures testable via primordial tensor modes and black-hole evaporation spectra.

## 1 Introduction

The GUFT programme re-imagines gravity as an emergent phenomenon: virtual gravitons in a quantised background accumulate coherently because no antimass state exists, generating curvature that mimics dark-matter phenomenology [1]. A prerequisite for predictive power, however, is a rigorous action principle that unifies geometry, quantum fields, and vacuum structure. This paper supplies that foundation.

## 2 Motivation and Guiding Principles

Any viable GUFT action must satisfy four criteria:

1. **Background independence:** spacetime geometry arises dynamically from field condensates.
2. **Low-energy correspondence:** for $|h_{\mu\nu}| \ll 1$ and constant DEQF, the equations reduce to Einstein–Hilbert gravity with cosmological constant $\Lambda$ and minimally coupled Standard Model.
3. **Ultraviolet finiteness:** Planck-scale saturation of graviton modes must regularise quantum loops.
4. **Symmetry compatibility:** internal gauge symmetries of the Standard Model must survive as coherent defect structures in the DEQF (addressed in a companion paper [2]).

## 3 The GUFT Action

We postulate

$$S = \int d^4x\,\sqrt{-g}\Big[\tfrac{1}{2\kappa}(R-2\Lambda) + \mathcal{L}_\Phi + \mathcal{L}_{\rm GQF} + \mathcal{L}_{\rm int}\Big], \tag{1}$$

with $\kappa = 8\pi G$. The individual pieces are:

$$\mathcal{L}_\Phi = -\tfrac{1}{2} g^{\mu\nu}\partial_\mu\Phi\,\partial_\nu\Phi - V(\Phi), \tag{2}$$

$$\mathcal{L}_{\rm GQF} = -\tfrac{1}{2}\nabla^\lambda h^{\mu\nu}\nabla_\lambda h_{\mu\nu} + \nabla^\mu h_{\mu\nu}\nabla_\lambda h^{\lambda\nu} - \nabla^\mu h\,\nabla^\nu h_{\mu\nu} + \tfrac{1}{2}\nabla^\lambda h\,\nabla_\lambda h, \tag{3}$$

$$\mathcal{L}_{\rm int} = \alpha\,\Phi\,T^{\mu\nu}(h) g_{\mu\nu}, \tag{4}$$

where $h = h^\mu{}_\mu$ and $T^{\mu\nu}(h)$ is the graviton stress–energy at quadratic order. A "Mexican-hat" potential $V(\Phi) = \lambda(\Phi^2 - v^2)^2$ enforces a vacuum expectation value $\langle\Phi\rangle = v$ whose fluctuations source $\Lambda$.

## 4 DEQF Condensate and Effective Geometry

Writing $\Phi = v + \varphi$ separates classical condensate from excitations. The effective background metric is then determined by solving

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa\big[T^{\rm matter}_{\mu\nu} + T^{\varphi}_{\mu\nu} + T^{\rm vg}_{\mu\nu}\big], \tag{5}$$

with $T^{\rm vg}_{\mu\nu}$ the vacuum stress of virtual gravitons [1]. Small in the Solar System, $T^{\rm vg}_{\mu\nu} \propto r^{-2}$ dominates on galactic scales.

## 5 Graviton Field Quantization

We expand the metric about $\eta_{\mu\nu}$: $g_{\mu\nu} = \eta_{\mu\nu} + \kappa^{1/2} h_{\mu\nu}$. In transverse-traceless (TT) gauge the canonical commutator reads

$$[\hat{h}_{ij}(\mathbf{x},t),\,\partial_0\hat{h}_{kl}(\mathbf{y},t)] = i\,\hbar\,\delta^3(\mathbf{x}-\mathbf{y})\,\Pi_{ijkl}, \tag{6}$$

with $\Pi_{ijkl}$ the TT projector. Mode expansion:

$$\hat{h}_{ij}(x) = \sum_{\vec{k},s} \frac{\sqrt{\hbar}}{\sqrt{2V\omega_k}}\Big(\epsilon^{(s)}_{ij} a_{\vec{k}s} e^{ikx} + \epsilon^{(s)*}_{ij} a^\dagger_{\vec{k}s} e^{-ikx}\Big), \tag{7}$$

$\omega_k$ obeying the saturation-modified dispersion (§6).

## 6 Planck-Scale Saturation and UV Convergence

We impose

$$\omega_k = \frac{|\vec{k}|}{\sqrt{1 + (|\vec{k}|/k_{\rm P})^2}}, \quad k_{\rm P} = \ell_{\rm P}^{-1}, \tag{8}$$

so $\omega_k \to \ell_{\rm P}^{-1}$ for $|\vec{k}| \to \infty$. One-loop graviton self-energy then converges:

$$\Pi(q^2) \sim \int_0^\infty dk\; k^2 \frac{\omega_k}{\omega_k^2 - q^2} < \infty. \tag{9}$$

## 7 Classical Limit and Standard Model Coupling

Setting $h_{\mu\nu} \to 0$ and $\varphi \to 0$ gives $S \to \int \sqrt{-g}(R - 2\Lambda)/2\kappa$, recovering GR. Standard Model fields couple minimally to $g_{\mu\nu}$; additional DEQF-induced operators are suppressed by $\ell_{\mathrm{P}}$, preserving known physics at accessible energies.

## 8 Conclusion

We have provided a consistent GUFT action, explicit DEQF potential, and canonical graviton quantisation with Planck-scale saturation. This framework simultaneously yields GR + SM in the IR and finite quantum gravity in the UV, laying groundwork for future calculation of cosmological perturbations and black-hole interiors.

## References

## References


[1] J. S. Trimm, "Virtual Graviton Density and the Dark Matter Phenomenon," Self-published (2025).

[2] J. S. Trimm, "Standard Model Fields as Emergent Excitations of Spacetime," Self-published (2025).